

### Charge

**Quantization of charge**  
 $Q = \pm ne$      $Q = \text{Total charge}$   
 $n = 1, 2, 3, \dots$   
 $e = 1.6 \times 10^{-19} \text{C}$

**Additivity of charge**  
 $Q' = Q_1 + Q_2$

**Redistribution of charge**

$Q' = \frac{Q_1 + Q_2}{2}$

$Q' = \text{Charge on each shell after redistribution}$

**Charge Density**

Linear Charge density,  $\lambda = \frac{Q}{L}$     Unit:  $\frac{\text{C}}{\text{m}}$   
 Surface Charge density,  $\sigma = \frac{Q}{S}$     Unit:  $\frac{\text{C}}{\text{m}^2}$   
 Volume Charge density,  $\rho = \frac{Q}{V}$     Unit:  $\frac{\text{C}}{\text{m}^3}$

$Q = \text{Total charge}$      $V = \text{Volume}$   
 $L = \text{Length}$      $S = \text{Area}$

**?** If a charge on the body is 1 nC, then how many electrons are present on the body?  
 a)  $1.6 \times 10^{19}$     b)  $6.25 \times 10^9$   
 c)  $6.25 \times 10^{27}$     d)  $6.25 \times 10^{18}$

### Coulomb's Law

$Q_1 \xrightarrow{F} Q_2$      $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$   
 $\epsilon_0 = \text{Permittivity of free space}$

$[\epsilon_0] = \frac{[Q_1][Q_2]}{[F]r^2} = \frac{[AT][AT]}{[LT^{-2}][L^2]} = M^{-1}L^{-2}T^2A^2$

$Q_1 \xrightarrow{F} Q_2$      $F_{\text{net}} = \frac{F}{k}$

$k = \text{dielectric constant of the medium}$

**Superposition**

**Direction:**  
 a) Like - Towards the point at which force has to be evaluated (repulsion)  
 b) Unlike - Away from the point at which force has to be evaluated (attraction)

**General rule**  
 $F_{\text{net}} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$

**When  $\theta = 60^\circ$**   
 $F_{\text{net}} = \sqrt{3}F$

**When  $\theta = 90^\circ$**   
 $F_{\text{net}} = \sqrt{2}F$

**When  $\theta = 120^\circ$**   
 $F_{\text{net}} = F$

### Equilibrium of Charges

**Calculation of Charge**

$Q_1, r_1, r_2, Q_2$

$Q_2 = \left(\frac{r_1}{r_2}\right)^2 q$  in equilibrium

$q = -\left(\frac{r_1}{r_1 + r_2}\right) Q_2$      $Q_1$  in equilibrium

$q = -\left(\frac{r_2}{r_1 + r_2}\right) Q_1$      $Q_2$  in equilibrium

**?** A charge is placed at the centre of the line joining two equal charges Q. The system of the three charges will be in equilibrium if q is equal to

a)  $-Q/2$     c)  $+Q/4$   
 b)  $-Q/4$     d)  $+Q/2$

### Charge on pendulum

$\tan\theta = \frac{qE}{mg}$

$\sin\theta = \frac{r}{2l}$

$r = 2l \sin\theta$

if  $\theta$  is very small  
 $\tan\theta \approx \sin\theta$

$\frac{r}{2l} = \frac{qE}{mg}$

$\frac{r}{2l} = \frac{kq^2/r^2}{mg}$      $r^3 \propto q^2$

Density of ball "P"

it  $\theta$  does not change on submerging in liquid  
 Dielectric constant of liquid,

$K = \frac{\rho}{\rho - \sigma}$

density of liquid =  $\sigma$

### Electric Field

Electric field at a point, due to point charge  $E = \frac{kq}{r^2}$

$K = \frac{1}{4\pi\epsilon_0}$

**Superposition**

General rule  
 $E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos\theta}$

$E_1 \rightarrow E_{\text{net}} = E_1 + E_2$

$E_2 \rightarrow E_{\text{net}} = |E_1 - E_2|$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos\theta}$   
 If,  $E_1 = E_2 = E$  Then,  $E_{\text{net}} = \sqrt{3}E$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2}$   
 If,  $E_1 = E_2 = E$  Then,  $E_{\text{net}} = \sqrt{2}E$

$E_{\text{net}} = \sqrt{E_1^2 + E_2^2 - 2E_1 E_2 \cos\theta}$   
 If,  $E_1 = E_2 = E$  Then,  $E_{\text{net}} = E$

**Direction**

1) Positive charge: - Towards the point at which electric field has to be evaluated  
 2) Negative charge: - Away from the point at which electric field has to be evaluated

### Neutral Point

**Like Charges**

$x_1 = \frac{|Q_1| r}{|Q_1| + |Q_2|}$

$x_2 = \frac{|Q_2| r}{|Q_1| + |Q_2|}$

$|Q_1| < |Q_2|$

Distance from  $Q_1 = x+r$

**?** Two point charges  $+8q$  and  $-2q$  are located at  $x = 0$  and  $x = L$  respectively. The location of a point on the x axis at which the net electric field due to these two point charges is zero is:

a) 8L  
 b) 4L  
 c) 2L  
 d) L/4

### Charged particle released in an electric field

1) Force,  $F = qE$   
 2) Acceleration,  $a = \frac{qE}{m}$   
 3) Velocity,  $V = \frac{qE}{m} t$   
 4) Velocity,  $V = \sqrt{2qE \frac{x}{m}}$   
 5) Kinetic energy,  $K.E = \frac{q^2 E^2 t^2}{2m}$

$V = \sqrt{V_x^2 + V_y^2}$   
 $= \sqrt{U^2 + \left(\frac{qE}{m} t\right)^2}$

accelerated in the direction of electric field

accelerated opposite to the direction of electric field

accelerated in the direction of field and perpendicular to initial velocity

$\frac{M_e}{M_p} = 1837$      $\frac{e}{m} = 1.7 \times 10^{11} \frac{1}{\text{s}}$   
 $\frac{1}{2} at^2 = \text{Constant}$

$t_1 = h$      $t_2 = \frac{1}{2} \frac{qE}{m} t^2 = h$   
 $t_2 = \sqrt{\frac{2m}{qE}} h^{1/2}$   
 $\Rightarrow t_1 > t_2$

### Time period of Charged Pendulum in an electric field

$T = 2\pi \sqrt{\frac{l}{g - \frac{QE}{m}}}$   
 Time period will increase

$T = 2\pi \sqrt{\frac{l}{g + \frac{QE}{m}}}$   
 Time period will decrease

$T = 2\pi \sqrt{\frac{l}{g^2 + \left(\frac{QE}{m}\right)^2}}$   
 Time period will decrease

**Electric field inside a dielectric medium**

$E_{\text{net}} = \frac{E}{k} \Rightarrow K$

$E_{\text{net}} = E - E_{\text{induced}}$      $E_{\text{induced}} = E - E_{\text{net}} = E\left(1 - \frac{1}{k}\right)$

### Properties of field lines

- Start from positive charge and end on negative charge
- Never intersect each other. If they intersect there will be 2 directions for electric field which is not possible
- Always perpendicular to Conducting surface
- $E \propto$  Electric field line density
- Never form closed loops (Conservative force)
- $q \propto$  no. of field lines  
 $|q_1| > |q_2|$

**?** Electric lines of force about negative point charge are:  
 a) circular, anticlockwise  
 b) circular, clockwise  
 c) radial, inward  
 d) radial, outward

### Electric flux

Flux is proportional to total no. of field lines passing through an area  
 $\Phi = \int E \cdot ds \cos\theta$   
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**Gauss Law:** -  $\Phi = \frac{q}{\epsilon_0} = \oint E \cdot ds \cos\theta$

**Zero flux:** -  $\Phi = \frac{q_{\text{net}}}{\epsilon_0} = 0$ , where  $q_{\text{net}} = 0$

**Electric flux for Cube**

- No charge inside the cube  
 $\Phi = \frac{q}{\epsilon_0} = 0$
- Charge placed at the center  
 $\Phi_{\text{total}} = \frac{q}{\epsilon_0}$   
 $\Phi_{\text{one side}} = \frac{q}{6\epsilon_0}$
- Charge placed at the edge  
 $\Phi_{\text{total}} = \frac{q}{\epsilon_0}$   
 $\Phi_{\text{one face}} = \frac{q}{8\epsilon_0} = \frac{1}{3} \times \frac{q}{24\epsilon_0}$
- Charge placed at the face  
 $\Phi_{\text{cube}} = \frac{q}{2\epsilon_0}$
- Charge placed at the corner  
 $\Phi_{\text{cube}} = \frac{q}{8\epsilon_0}$   
 $\Phi_{\text{one face}} = \frac{q}{24\epsilon_0} = \frac{1}{3} \times \frac{q}{72\epsilon_0}$
- Flux through curved surface  
 $\Phi_{\text{effective}} = \Phi_{\text{upper}} + 2\Phi_{\text{cross section}}$

### Application of Gauss's Theorem

- Point charge  $E = \frac{kq}{r^2}$
- Metal sphere/Hollow sphere  
 $E_{\text{surface}} = \frac{kQ}{R^2}$   
 $E_{\text{outside}} = \frac{kQ}{r^2}$   
 $E_{\text{inside}} = 0$
- Non-Conducting sphere  
 $E_{\text{inside}} = \frac{kQr}{R^3}$   
 $E_{\text{surface}} = \frac{kQ}{R^2}$   
 $E_{\text{outside}} = \frac{kQ}{r^2}$
- Conducting sheet  
 $E = \frac{\sigma}{2\epsilon_0}$
- Non-conducting sheet  
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